# Bounds on the Maximum Number of Concurrent Links in MIMO Ad Hoc Networks with QoS Constraints

Pengkai Zhao, and Babak Daneshrad

Wireless Integrated Systems Research (WISR) Group, Electrical Engineering Department,

University of California, Los Angeles, CA 90095 USA

(e-mail: pengkai@ee.ucla.edu; babak@ee.ucla.edu)

**Abstract**

Multiple-Input Multiple-Output (MIMO) based Medium Access Control (MAC) protocols have received a good deal of attention as researchers look to enhance overall performance of Ad Hoc networks by leveraging multi antenna enabled nodes [1]–[5]. To date such MAC protocols have been evaluated through comparative simulation based studies that report on the number of concurrent links the protocol can support. However, a bound on the maximum number of concurrent links (MNCL) that a MIMO based MAC protocol should strive to achieve has hitherto been unavailable. In this paper we present a theoretical formulation for calculating the bound on the MNCL in a Mobile Ad Hoc Network (MANET) where the nodes have multiple antenna capability, while guaranteeing a minimum Quality of Service (QoS). In an attempt to make our findings as practical and realistic as possible, the study incorporates models for the following PHY layer and channel dependent elements: (a) path loss and fast fading effects, in order to accurately model adjacent link interference; (b) a Minimum Mean Squared Error (MMSE) based detector in the receiver which provides a balance between completely nulling of neighboring interference and hardware complexity. In calculating the bound on the MNCL our work also delivers the optimal power control solution for the network as well as the optimal link selection. The results are readily applicable to MIMO systems using Receive Diversity, Space Time Block Coding (STBC), and Transmit Beamforming and show that with a 4 element antenna system, as much as $3\times$ improvement in the total number of concurrent links can be achieved relative to a SISO based network. The results also show diminishing improvement as the number of antennas is increased beyond 4, and the maximum allowable transmit power is increased beyond 10 dBm (for the simulated parameters).

**Index Terms**

MIMO, MANET, MAC, MMSE, Concurrent Links, Receive Diversity, STBC, Beamforming.

## I. Introduction

Wireless Ad Hoc networking has emerged as an important aspect of next generation communication systems. For conventional Single-Input Single-Output (SISO) system, interference among nodes drastically limits the number of concurrent (simultaneous) links in Ad Hoc networks. Multi antenna, multi-input-multi-output (MIMO), based wireless communications has the ability to spatially null interference and in so doing increase the number of concurrent links within a Mobile Ad Hoc Networks (MANET), thus increasing overall network throughput. In fact some work found in the literature [1]–[6] look to MIMO capable MAC protocols as a means of increasing the network efficiency and its sum-throughput.

The maximum number of concurrent links is a metric used in the literature [1], [7], [8] to evaluate the capacity of a network. Examples of MACs that support concurrent links in a network where all nodes have multiple antennas at their disposal can be found in [1]–[5], [7]. For convenience these MAC protocols will be referred to as concurrent-based MACs in this paper. The Null-Hoc [2] and SPACE-MAC [3] protocols look to enable concurrent links by using the Gram-Schmidt Orthonormalization, so as to create orthogonal channels among different links. The protocol in [4] uses adaptive interference cancellation both at the transmitter and at the receiver, as well as a simple power control protocol for each link. Multiple links are assumed to access the channel sequentially and work simultaneously. The MIMA-MAC protocol [5] uses space division multiple access techniques to guarantee the concurrency of different communicating links in the network.

Although these concurrent-based MAC protocols have proved to outperform the conventional SISO based MACs such as the IEEE 802.11 DCF [9], a natural question to ask is that how close they actually come to the theoretical bound (limit) of concurrency in Ad Hoc networks. Furthermore, since MIMO systems enable a variety of approaches in utilizing multiple antennas in the physical layer [10], it is also concerned that how this bound of concurrency is affected by the choice of MIMO algorithms and associated physical layer techniques. In this work we identify the theoretical Maximum Number of Concurrent Links (MNCL) in the network by considering

the following PHY layer and channel dependent elements: (a) path loss and fast fading effects; (b) different MIMO transmit/receive algorithms; (c) a Minimum Mean Squared Error (MMSE) based detector in the receiver; (d) optimal power control and optimal link selection. The derived MNCL acts as a performance benchmark for concurrent-based MAC protocols, and is also used as a metric for comparing different MIMO techniques and parameters.

Our study is based on the assumption that each transmit/receive pair requires the same Constant Bit Rate (CBR). In this way, the MNCL that can be had in a MIMO capable network is identified subject to a minimum Quality of Service (QoS) constraint. In our case the QoS constraint is the received Signal to Interference plus Noise Ratio (SINR) which is directly related to Bit Error Rate (BER) and Packet Error Rate (PER). The proposed framework is executed via an iterative process for power allocation and a Backtracking-based search strategy for link selection. Results of different MIMO transmit algorithms such as Receive Diversity, Space-Time Block Coding (STBC) and Transmit Beamforming on the MNCL bound are studied. Using this bound we present design-relevant insight regarding the impact of the number of pairs, the number of antennas, and the maximum allowable transmit power per pair on the MNCL.

The reminder of this paper is organized as follows. The proposed system model is described in Section II. The definition of concurrent links is provided in Section III. Optimal power allocation is investigated in Section IV. In Section V, we propose a backtracking-based strategy to find the optimal link selection. Numerical results are shown in Section VI, and we conclude in Section VII.

## II. SYSTEM MODEL

### A. Network Model

We focus on a small network (or sub-network), where each node is within the transmission range of any other node subject to path loss and Rayleigh flat fading (Fig. 1). Assume $K$ simultaneously communicating pairs in the network. Each pair is composed of one transmit node and one receive node, which are all randomly distributed in the network. Nodes in different pairs are unique and independent. They are all equipped with $M$ receive antennas, sharing the same frequency band and requiring the same Constant Bit Rate (CBR). The number of transmit antennas varies depending on the MIMO technique being considered. For simplicity,

each communicating pair is labeled as a *Transceiver Pair*. For the $k$th transceiver pair, the transmit node and receive node are named *Tx Node $k$* and *Rx Node $k$*, respectively.

The propagation between nodes is characterized by path loss and Rayleigh fading [11]–[13]. For path loss, we use the simplified model in [13], which is:

$$L_P(d)(\text{dB}) = L_P(d_0) + 10\alpha \log_{10} \frac{d}{d_0} \tag{1}$$

In this model, $\alpha$ is the path loss exponent, $d_0$ is the reference distance, and $d$ is the distance between nodes obtained from node topology information. Both $\alpha$ and $d_0$ are parameters in our study and can be set by the user. We use $\rho_{kj}$ to denote the power loss ratio from Tx Node $j$ to Rx Node $k$, and $\rho_{kj} = 10^{-\frac{L_P(d)}{10}}$.

We assume a flat fading environment, which is modeled by a Rayleigh distribution. $\mathbf{H}_{kj}(m)$ denotes the Rayleigh fading channel from the $m$th antenna of Tx Node $j$ to Rx Node $k$. It is an $M \times 1$ vector and consists of independent identically distributed (i.i.d.) complex Gaussian random variables.

The background white noise is a circularly complex Gaussian vector with covariance matrix $\sigma_N^2 \mathbf{I}_M$. $\mathbf{I}_M$ is an $M \times M$ unitary matrix, and $\sigma_N^2$ is given by:

$$\sigma_N^2(\text{dBm}) = \eta_n + 10\log_{10}(W) + F_n \tag{2}$$

$W$(Hz) is the bandwidth of the system, while $\eta_n$(dBm/Hz) and $F_n$(dB) are the power spectral density of the thermal noise and the noise figure of the receiver, respectively. They are assumed to be identical for all the nodes.

*B. Transceiver Model*

Three different MIMO techniques are considered in this paper, they are: (a) $1 \times M$ Receive Diversity; (b) $2 \times M$ Space-Time Block Coding (STBC); and (c) $M \times M$ Transmit Beamforming. In this paper, we assume that data symbols in the baseband have the same modulation type, regardless of single or multiple antennas. For fair comparison, the number of transmit antennas in these MIMO techniques are selected so that their spectral efficiency is the same as that of a Single-Input Single-Output (SISO) system.

*1) $1 \times M$ Receive Diversity:* In the $1 \times M$ scenario [14], only one transmit antenna is employed. Without loss of generality, the 1st transmit antenna is used. Considering the $k$th transceiver pair, the received signal at Rx Node $k$ is given by:

$$\mathbf{Y}_k = \sqrt{P_k \rho_{kk}} \mathbf{H}_{kk}(1) x_k + \mathbf{R}_k^{(SD)} + \mathbf{N}_k \tag{3}$$

$$\mathbf{R}_k^{(SD)} = \sum_{i=1, i \neq k}^{K} \sqrt{P_i \rho_{ki}} \mathbf{H}_{ki}(1) x_i \tag{4}$$

In these equations, $\mathbf{Y}$, $\mathbf{H}$, $\mathbf{R}$ and $\mathbf{N}$ are all $M \times 1$ vectors. $\mathbf{R}$ denotes the inter-pair interference. $x_k$ is the transmitted symbols for the $k$th pair, which has zero mean and unit variance. $P_k$ is the allocated power for $x_k$. $\mathbf{N}_k$ is a white Gaussian noise vector with covariance $\sigma_N^2 \mathbf{I}_M$.

*2) $2 \times M$ STBC:* In the $2 \times M$ Space-Time Block Coding (STBC) scenario we use an Alamouti Code [15] with two transmit antennas. We use $x_{k,m,n}$ to denote the transmitted symbol at $m$th antenna and $n$th time slot of the $k$th pair. Using the Alamouti Code, we have $x_{k,1,2} = -x_{k,2,1}^*$ and $x_{k,2,2} = x_{k,1,1}^*$, $(1 \leq m \leq 2, 1 \leq n \leq 2)$. Here $x_{k,1,1}$ and $x_{k,2,1}$ are independent symbols with zero mean and unit variance, and $(\cdot)^*$ is the complex conjugation. $P_k(1)$ and $P_k(2)$ are allocated power for $x_{k,1,1}$ and $x_{k,2,1}$, respectively. Assume the received vectors corresponding to these two time slots are $\mathbf{Y}_{k,1}$ and $\mathbf{Y}_{k,2}$, as well as the noise vectors $\mathbf{N}_{k,1}$ and $\mathbf{N}_{k,2}$. Define new vectors by $\overline{\mathbf{Y}}_k = \left[ \mathbf{Y}_{k,1}; \mathbf{Y}_{k,2}^*; \right]$, $\overline{\mathbf{H}}_{ki,1} = [\mathbf{H}_{ki}(1); \mathbf{H}_{ki}(2)^*;]$, $\overline{\mathbf{H}}_{ki,2} = [\mathbf{H}_{ki}(2); -\mathbf{H}_{ki}(1)^*;]$ and $\overline{\mathbf{N}}_k = [\mathbf{N}_{k,1}; \mathbf{N}_{k,2}^*;]$. Consequently, the received signal at Rx Node $k$ can be denoted as:

$$\overline{\mathbf{Y}}_k = \sqrt{P_k(1) \rho_{kk}} \overline{\mathbf{H}}_{kk,1} x_{k,1,1} + \sqrt{P_k(2) \rho_{kk}} \overline{\mathbf{H}}_{kk,2} x_{k,2,1} + \mathbf{R}_k^{(STBC)} + \mathbf{N}_k \tag{5}$$

$$\mathbf{R}_k^{(STBC)} = \sum_{i=1, i \neq k}^{K} \sum_{l=1}^{2} \sqrt{P_i(l) \rho_{ki}} \overline{\mathbf{H}}_{ki,l} x_{i,l,1} \tag{6}$$

*3) $M \times M$ Transmit Beamforming:* The Beamforming method outlined in [16], [17] is also employed in this paper. $M$ transmit antennas are adopted, and an $M \times 1$ weight vector $\mathbf{u}_k$ is applied to these transmit antennas. Let $\mathbf{H}_{kk} = [\mathbf{H}_{kk}(1), \mathbf{H}_{kk}(2), \ldots, \mathbf{H}_{kk}(M)]$, then according to [16], [17], $\mathbf{u}_k$ is the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{H}_{kk}^H \mathbf{H}_{kk}$, where $(\cdot)^H$ denotes conjugate transpose. We assume that $\mathbf{u}_k^H \mathbf{u}_k = 1$ and $x_k$ has zero mean and unit variance. The transmit power for $x_k$ is $P_k$. Then the received signal under this method is represented as:

$$\mathbf{Y}_k = \sqrt{P_k \rho_{kk}} \mathbf{H}_{kk} \mathbf{u}_k x_k + \mathbf{R}_k^{(Beam)} + \mathbf{N}_k \tag{7}$$

$$\mathbf{R}_k^{(Beam)} = \sum_{i=1, i \neq k}^{K} \sqrt{P_i \rho_{ki}} \mathbf{H}_{ki} \mathbf{u}_i x_i \tag{8}$$

*C. MMSE Solution*

For all three scenarios listed in the previous section we will use the Minimum Mean Squared Error (MMSE) solution at the receiver [18], [19], [21] to arrive at the receive MIMO antenna weights. Here we only present the MMSE solution for the $1 \times M$ Receive Diversity case. However, the MMSE solution can also be derived for the $2 \times M$ STBC and $M \times M$ Transmit Beamforming scenarios by slight modification (the associated results are given in Appendix I).

For the $1 \times M$ Receive Diversity scenario, the estimate of the transmitted stream $x_k$ at the receiver is given by (9) where the vector $\mathbf{w}_k$ is the MMSE solution.

$$\hat{x}_k = \mathbf{w}_k^H \mathbf{Y}_k \tag{9}$$

The corresponding SINR at the receiver is:

$$\Gamma_k = \frac{P_k \rho_{kk} \mathbf{w}_k^H \mathbf{H}_{kk}(1) \mathbf{H}_{kk}^H(1) \mathbf{w}_k}{\mathbf{w}_k^H \Phi_{(SD)}(k) \mathbf{w}_k} \tag{10}$$

$$\Phi_{(SD)}(k) = \sum_{1 \leq i \leq K, i \neq k} P_i \rho_{ki} \mathbf{H}_{ki}(1) \mathbf{H}_{ki}^H(1) + \sigma_N^2 \mathbf{I}_{\mathrm{M}} \tag{11}$$

With the constraint that $\mathbf{w}_k^H \mathbf{H}_{kk}(1) = 1$, optimal linear vector $\mathbf{w}_k$ in the MMSE solution is given in [18], [19], [21]

$$\mathbf{w}_k = \frac{\Phi_{(SD)}^{-1}(k) \mathbf{H}_{kk}(1)}{\mathbf{H}_{kk}^H(1) \Phi_{(SD)}^{-1}(k) \mathbf{H}_{kk}(1)} \tag{12}$$

The SINR with MMSE solution $\hat{\Gamma}_k$ is:

$$\hat{\Gamma}_k = P_k \rho_{kk} \mathbf{H}_{kk}^H(1) \Phi_{(SD)}^{-1}(k) \mathbf{H}_{kk}(1) \tag{13}$$

Finally, we assume a QoS requirement at the receiver. Consider an SINR threshold $\gamma_T$, for the $k$th transceiver pair, they can be correctly received if and only if the received SINR is not lower than $\gamma_T$. The threshold $\gamma_T$ represents the QoS requirement and is being used here in the same manner as the Physical Model in [20].

## III. The Definition of Concurrent Links

We assume that all Tx and Rx Nodes are mobile, and their locations are randomly distributed and varying with time. In the following discussion, a specific scenario means one specific realization of node locations in the region, and the channel responses between them. For a specific scenario, given the allocated power in each pair, the SINR for each pair can be evaluated using the results of Section II. Based on these SINR results, we first provide the following definition.

*Definition 1 (Link):* Consider a specific scenario with $K$ transceiver pairs. The transmit power for every pair is constrained by $P_T$. The $k$th pair $(1 \leq k \leq K)$ is called a *link* if and only if it satisfies the following conditions:

For $1 \times M$ Receive Diversity and $M \times M$ Transmit Beamforming:

$$\hat{\Gamma}_k \geq \gamma_T \text{ and } P_k \leq P_T \tag{14}$$

For Space Time Block Coding:

$$\hat{\Gamma}_k(l) \geq \gamma_T, 1 \leq l \leq 2 \text{ and } \sum_{l=1}^{2} P_k(l) \leq P_T \tag{15}$$

Assume a specific scenario with $K$ transceiver pairs, labeled from $1$ to $K$ and denoted by a set $U = \{1, 2, \ldots, K\}$. A pair set $U_P \subseteq U$ is a feasible pair set if there exists a power allocation for all transmitters in $U_P$ such that *Definition 1* holds for all transceiver pairs in $U_P$ (i.e., the QoS constraint is satisfied at all receivers subject to the maximum $P_T$ constraint). Then all the pairs in $U_P$ are named *Concurrent Links*.

Let us denote the number of pairs in $U_P$ as: $|U_P|$. Then the MNCL is calculated by the following optimization problem:

$$\max \quad |U_P|$$
$$\text{s.t.} \quad U_P \subseteq U \text{ and } U_P \text{ is feasible} \tag{16}$$

For notational simplicity, the result of the above optimization problem is denoted as $N_{\max}(K, M)$. Note that several different feasible sets $U_P$ may produce the same $N_{\max}(K, M)$. We then define *Average MNCL* as the expectation of $N_{\max}(K, M)$, averaged over random locations and random channel responses. The Average MNCL will be denoted as $C(K, M)$ in this paper.

The optimization in equation (16) can be divided into two steps: 1) to examine whether a pair vector $U_P$ is feasible; and 2) to search for the feasible pair set $U_P$ with the MNCL. In this paper, the first step is solved by an optimal power allocation process presented in Section IV, while the second step, referred to as optimal link selection, is solved in Section V.

## IV. OPTIMAL POWER ALLOCATION

Optimal power allocation is an important factor when deriving the bound on the MNCL. In our formulation it is used to decide whether a given pair set $U_P$ is feasible per *Definition 1*. In this section, the algorithm for optimal power allocation is presented from conventional power control techniques [21]–[23]. Without loss of generality, only the $1 \times M$ Receive Diversity scenario is considered in this section. However, the proposed algorithm can also be applied to the $2 \times M$ STBC and $M \times M$ Transmit Beamforming scenarios after slight modification.

### A. Iterative Power Control

Consider $K$ transceiver pairs in the network using $1 \times M$ Receive Diversity. The allocated power for each pair is stacked into a vector $\mathbf{P}$, which is defined as the power vector and given by:

$$\mathbf{P} = [P_1, P_2, \ldots, P_K] \tag{17}$$

Since we only focus on pairs in $U_P$, we have the following constraint:

$$\text{for } k \notin U_P, \ P_k = 0, \ 1 \leq k \leq K \tag{18}$$

Let $\mathbf{w}_k$ be the MMSE solution in equation (12). We define the following iteration equation: For $k \in U_P$

$$P_k^{n+1} = \gamma_T \frac{C_k^{(SD)} \{\mathbf{w}_k, \mathbf{P}^n\} + \sigma_N^2 \mathbf{w}_k^H \mathbf{w}_k}{\rho_{kk}} \tag{19}$$

$$C_k^{(SD)} \{\mathbf{w}_k, \mathbf{P}^n\} = \sum_{i=1, i \neq k}^{K} P_i^n \rho_{ki} G \{\mathbf{w}_k, \mathbf{H}_{ki}(1)\} \tag{20}$$

$$G \{\mathbf{w}_k, \mathbf{H}_{ki}(1)\} = \mathbf{w}_k^H \mathbf{H}_{ki}(1) \mathbf{H}_{ki}^H(1) \mathbf{w}_k \tag{21}$$

For $1 \leq k \leq K$ and $k \notin U_P$

$$P_k^{n+1} = 0 \tag{22}$$

where $n$ denotes the $n$th iteration. For simplicity, the above iteration is denoted as

$$\mathbf{P}^{n+1} = m\left(\mathbf{P}^n\right) \tag{23}$$

Here $\mathbf{P}^n$ is the power vector for the $n$th iteration.

We define the fixed point of mapping as the power vector $\widehat{\mathbf{P}}$ satisfying $\widehat{\mathbf{P}} = m(\widehat{\mathbf{P}})$. The following theorem holds for the iterative equation (23), which is used to verify the existence of optimal power allocation in this paper.

*Theorem 1:* Given $K$ transceiver pairs and a specific pair set $U_P$. If $U_P$ is feasible, then a unique fixed point of mapping, $\widehat{\mathbf{P}}$, exists that satisfies $\widehat{\mathbf{P}} = m(\widehat{\mathbf{P}})$ and $\widehat{P}_k \leq P_T, \forall k \in U_P$. Furthermore, corresponding to the unique power vector $\widehat{\mathbf{P}}$ is a unique receive weight vector $\widehat{\mathbf{w}}_k$ given by the MMSE solution.

*Proof:* Please refer to Appendix II. Note that the converse proposition of this theorem holds obviously. That is, if there exists $\widehat{\mathbf{P}} = m(\widehat{\mathbf{P}})$ and $\widehat{P}_k \leq P_T, \forall k \in U_P$, then $U_P$ is feasible. ∎

### B. Decision Criteria

Theorem 1 suggests that a feasible pair set can be identified if and only if a fixed point of mapping for the transmit power levels exists and the power constraints are met. Moreover, [21] suggests that the power control algorithm in (23) is guaranteed to converge to a fixed point of mapping even when starting from an arbitrary initial power vector $\mathbf{P}^0$. This suggests a rather straight forward approach for determining the feasibility of a given pair set $U_P$ as captured by the following three criteria.

*Criterion 1:* Assume that the iteration process starts from initial condition $\mathbf{P}^0 = 0$. In each iteration, if the power in any transceiver pair exceeds the power constraint $P_T$, then $U_P$ is not feasible.

The proof of Criterion 1 is referred to Theorem 2 in [21].

*Criterion 2:* Assume that the iteration process starts from arbitrary condition $\mathbf{P}^0$. In each iteration, if $\mathbf{P}^n$ is feasible, then these pairs can be supported simultaneously.

Here, saying $\mathbf{P}^n$ is feasible means that by using $\mathbf{P}^n$, for $1 \le k \le K$, we can have $\hat{\Gamma}_k \ge \gamma_T$ and $P_k^n \le P_T$. This criterion is straightforward and it is helpful in reducing the iterations in the decision process.

*Criterion 3:* After $I_{\max}$ number of iterations, if the conditions in *Criterion 2* have not yet been satisfied for pairs in $U_P$, then we declare that $U_P$ is not feasible.

Criterion 3 guarantees that the decision can be made within a finite iteration number $I_{\max}$. Using this criterion, some feasible pair sets may be missed. However, proper selection of $I_{\max}$ can drive the probability of missing a feasible pair set to be arbitrarily close to zero.

Using the above criteria we now outline a sequential procedure by which we can automatically determine if a set $U_P$ is feasible. The steps are as follows:

*Iterative Determination of Feasibility (IDF)*

1) Given pair set $U_P$. Initialize $n = 0$ and $\mathbf{P}^0 = 0$;
2) Iterate by $\mathbf{P}^{n+1} = m(\mathbf{P}^n)$. Using the updated power vector, $\mathbf{P}^{n+1}$, calculate the SINR for each pair under the MMSE solution;
3) If $U_P$ is not feasible by Criterion 1, then go to step 6; else, go to step 4;
4) If $U_P$ is feasible by Criterion 2, then go to step 6; else, go to step 5;
5) If $U_P$ is not feasible by Criterion 3, then go to step 6; else, $n = n + 1$, go to step 2.
6) The feasibility of $U_P$ is determined, stop.

## V. FINDING $U_P$ WITH MAXIMUM NUMBER OF CONCURRENT LINKS

In the previous section, we derived the method to determine if a pair set is feasible. In this section we describe how to find the feasible pair set with the MNCL. Naturally, a brute force search can be employed for this problem. However, in order to reduce the search space, we first characterize the property of the feasible pair set.

Consider a total of $K$ transceiver pairs, labeled 1 to $K$. Define $U = \{1, 2, \ldots, K\}$ and $U_P$ is a given pair set.

*Theorem 2:* For two pair sets $U_{P,2} \subseteq U_{P,1}$, if $U_{P,1}$ is feasible, then $U_{P,2}$ is also feasible.

*Proof:* Consider the $1 \times M$ Receive Diversity scenario. Let $(U_{P,1} - U_{P,2})$ denote the pairs in $U_{P,1}$ but not in $U_{P,2}$. If $U_{P,1}$ is feasible with the associated power vector $\mathbf{P}$, then for the pairs in $U_{P,2}$, keep the power $P_k$ and linear vector $\mathbf{w}_k$ unchanged. Meanwhile, shut down the pairs in $(U_{P,1} - U_{P,2})$. Here, *shutting down* means setting the corresponding transmit power to zero. As

a result, there are less interference in $\Phi_{(SD)}(k)$ of equation (10), and conditions in *Definition 1* are satisfied for the pairs in $U_{P,2}$. Thus $U_{P,2}$ is feasible and the above theorem holds. The proof is extendable for STBC and Beamforming methods as well. ∎

Theorem 2 shows that the backtracking formulation in [8], [24] can be adopted to solve the problem of finding the $U_P$ with the MNCL. We start this formulation with an empty pair set. At every level, each feasible subset is expanded by including one more pair, constructing new subsets to be validated by the IDF iterations presented in Section IV (namely, forward search). If one subset becomes unfeasible, the algorithm backtracks by removing the trailing pair from the subset, and then proceeds by expanding the subset with alternative pairs (namely, backward search). Specifically, we use a depth-first search strategy to execute this procedure. An example with 4 pairs are illustrated in Fig. 2, where all feasible pair sets are outlined. Here the initial feasible subsets are $\{\{1\}, \{2\}, \{3\}, \{4\}\}$, while the final feasible subsets are $\{\{1, 2, 3\}, \{1, 3\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{4\}\}$, and the MNCL is 3.

Let $U_{P,search}$ denote the pair set candidate during the search, and $|U_{P,search}|$ is the number of elements in $U_{P,search}$. Meanwhile, use $direction = 1$ to represent the forward search, and $direction = 0$ the backward search. The pseudocode of the proposed algorithm is described in the following.

*Backtracking-based Optimal Link Selection (BOLS)*

1) Given $K$ pairs (labeled from 1 to $K$) in the network and $M$ receive antennas per pair. Initialize by $U_{P,search} = \{1\}$, $N_{\max}(K, M) = 0$.

2) If $|U_{p,search}| > N_{\max}(K, M)$, then:

   a) Use the IDF process in Section IV to examine whether $U_{P,search}$ is feasible. If $U_{P,search}$ is feasible, then set $N_{\max}(K, M) = |U_{P,search}|$ and $direction = 1$; otherwise, set $direction = 0$.

3) If $|U_{p,search}| \leq N_{\max}(K, M)$, then set $direction = 1$.

4) Update the pair set candidate $U_{P,search}$:

   a) If $U_{P,search} = \{K\}$, then all the search have been done, return $N_{\max}(K, M)$ and stop.

   b) If $U_{P,search} \neq \{K\}$, then $U_{P,search} = \textbf{PairSet\_Gen}\,[U_{P,search}, direction]$ and go to step 2.

In step 3, we only examine the pair set $U_{P,search}$ in which $|U_{P,search}|$ is larger than the current

value of $N_{\max}(K, M)$. In other words, if $|U_{P,search}| \leq N_{\max}(K, M)$, instead of employing the IDF procedure, we set $direction = 1$ and go to step 4 directly. Next, the function PairSet_Gen is described as follows:

*Function* **PairSet_Gen**

1) Input $U_{P,search}$ and $direction$. Find $k_{\max} = \max\limits_{k \in U_{P,search}} k$, which represnts the pair with the maximum index in $U_{P,search}$.

2) If $k_{\max} < K$, then:

    a) If $direction = 1$, add new element in $U_{P,search}$ by $U_{P,search} = U_{P,search} \cup \{k_{\max} + 1\}$. Return $U_{P,search}$ and stop.

    b) If $direction = 0$, update the element $k_{\max}$ in $U_{P,search}$ by $k_{\max} = k_{\max} + 1$. Return $U_{P,search}$ and stop.

3) If $k_{\max} = K$ and $|U_{P,search}| > 1$, then;

    a) Delete $k_{\max}$ from $U_{P,search}$ by $U_{P,search}^{del} = U_{P,search} - \{k_{\max}\}$. Find $\overline{k}_{\max} = \max\limits_{k \in U_{P,search}^{del}} k$, update the element $\bar{k}_{\max}$ in $U_{P,search}^{del}$ by $\bar{k}_{\max} = \bar{k}_{\max} + 1$.

    b) Set $U_{P,search} = U_{P,search}^{del}$. Return $U_{P,search}$ and stop.

4) If $k_{\max} = K$ and $|U_{P,search}| = 1$, then all the search have been done. Return $U_{P,search}$ and stop.

Finally, averaging $N_{\max}(K, M)$ among random locations and random channel responses, we obtain the $C(K, M)$.

## VI. SIMULATION RESULTS

The simulation setup randomly distributes the nodes, in accordance to a uniform distribution, within a disk of radius 100 meters. Numerical results are averaged over 1000 Monte Carlo simulations of independent realizations of node topology and channel response. We assume QPSK modulation in the baseband, and the desired SINR threshold, $\gamma_T$, is 10dB. This corresponds to an uncoded BER of less than 1e-3 (see Table 6.1 in [14]). The remaining parameters are listed in Table I.

### A. Comparison of MIMO Techniques

In this section we evaluate the performance of the proposed algorithms, and use the MNCL as a metric to compare different MIMO methods. The three MIMO methods outlined in Section II are

compared, namely, $1 \times M$ Receive Diversity, $2 \times M$ STBC, and $M \times M$ Transmit Beamforming, all with MMSE decoding.

We assume up to 15 transceiver pairs, and vary the number of receive antennas from 1 to 4. Results for the three different MIMO methods are reported in Fig. 3 and Fig. 4. We observe that Transmit Beamforming has the best performance due to the fact that it explores Channel State Information (CSI) at the transmitter. On the other hand, STBC has a worse performance than Receive Diversity. Note that STBC is conventionally designed to cope with the white Gaussian noise [15], but in this study we use MIMO system to combat the colored interference signals (equation (11)), and our results indicate that Receive Diversity outperforms STBC in this case. Finally, performance of conventional SISO system corresponds to that of Receive Diversity with $M = 1$. We observe that compared to a SISO system, as much as $3\times$ improvement in MNCL can be achieved by using $M = 4$ receive antennas (compare the highlighted values for $K = 5, 10, 15$ in Fig. 3 with those in Fig. 4).

Next, we study the convergence of the estimate for the MNCL. We assume a total of 12 transceiver pairs in the network, with the number of receive antennas fixed 4. As previously mentioned, we create a realization by distributing the pairs in a 100m radius disk and randomly generating the channel responses among them. In Fig. 5, we plot the average number of MNCL versus the number of realizations. We find that the simulation results become convergent after around 500 realizations. Note that we used 1000 independent realizations in these simulations, which is large enough to yield a precise estimate for the performance of the proposed algorithm.

Finally, we compare the proposed backtracking strategy (Section V) with the brute force search. We have verified that, for $1 \leq K \leq 15$, the proposed backtracking method has exactly the same MNCL as the brute force search (the results are omitted due to space limit). Here we focus on the search complexity of these two search methods, that is, the number of times the IDF iterations were called. The average result over 1000 Monte Carlo simulations is shown in Fig. 6. Note that the associated complexity for the brute force method is $2^K$, where $K$ is the number of pairs in the network. Compared with the brute force search, we observe a significant reduction in complexity when the backtracking scheme is used. This verifies the efficacy of our proposed scheme.

*B. Impact of Simulation Parameters*

Using the formulation developed in this work, we can investigate the impact of different parameters, such as the number of transceiver pairs, the number of receive antennas, and the maximum allowable transmit power on the MNCLs. We first look at the number of pairs in the network. Results in Fig. 3 and Fig. 4 have already shown that the MNCL increases substantially with the number of pairs. Actually, we can use a two-stage linear equation to approximate the result of the MNCL. The equation is presented in (24), and the approximation result for Transmit Beamforming is demonstrated in Fig. 7 and Table II.

For $1 \leq K \leq 15$ ($a_1, b_1, a_2, b_2$ are parameters to be fitted):

$$C(K, M) = \begin{cases} a_1 + b_1 K, & \text{if } 1 \leq K \leq M \\ a_2 + b_2 K, & \text{if } M + 1 \leq K \leq 15 \end{cases} \qquad (24)$$

In the above equation, when $1 \leq K \leq M$, the improvement is mainly from the diversity gain; while when $M + 1 \leq K \leq 15$, the improvement is from multi-user diversity (as the number of pairs wanting to communicate increases the likelihood of choosing a subset of these pairs that exhibit good interference properties increases). The results in Fig. 3 and Fig. 4 show that the MNCL is larger than $M$ when the number of pairs $K$ is increased beyond $M$. This observation highlights some extra gains in MNCL to be exploited by the MAC protocol.

Next, we focus on the impact of the number of receive antennas. We assume 12 transceiver pairs, and the number of receive antennas $M$ varies from 1 to 8. Simulation results are shown in Fig. 8. We see that in this simulation, the MNCL is improved dramatically with the number of receive antennas. Again, we find that with 4 receive antennas, the improvement is around 3 times relative to the SISO system. Furthermore, we observe that saturation starts to set in when the number of receive antennas is greater than 5. With 8 receive antennas the gain relative to a SISO system is $4\times$.

Lastly, we explore the impact of the maximum allowable transmit power $P_T$. Naturally, higher transmit power is useful to combat path loss and increase transmission range. However, here we analyze the impact of $P_T$ under the assumption that all the nodes are distributed in a fixed disk with radius 100m. This assumption is reasonable which corresponds to a geographically constrained area with many potential transmission pairs (e.g., classroom, conference room). We assume a total of 10 pairs in the network, with 4 receive antennas per pair, and consider different

power constraints from -20dBm to 50dBm. The results are depicted in Fig. 9. We observe that for each MIMO method, initially, the MNCL increases with higher power constraint. However, when $P_T$ is beyond 10dBm, the improvement becomes diminishing. The reason is that co-channel interference among pairs has dominated the network, and increasing $P_T$ can not mitigate these interfering signals.

## VII. CONCLUSION

In this paper, we have identified the bound on the maximum number of concurrent links (MNCL) in MIMO Ad Hoc networks subject to a minimum QoS (SINR in our case) for each link. This number is derived by considering the practical factors of wireless channel, transmit power allocation, link selection and MIMO transceiver algorithms in a realistic MIMO system. We employ an iterative algorithm to examine the existence of an optimal power allocation that guarantees QoS for all pairs in a given set. Based on this iterative algorithm, we proposed a backtracking search algorithm to select the optimal subset of pairs, constituting the MNCL.

Extensive simulations were conducted to verify the efficacy of the proposed algorithms and evaluate the impact from different parameters. The results show a $3\times$ improvement in MNCL with a 4 element antenna system relative to a SISO system. For the parameters simulated, diminishing improvement is observed when the number of antennas is increased beyond 5, and when the maximum transmit power is increased beyond 10dBm.

## APPENDIX A

In this appendix, we show the MMSE solution for STBC and Transmit Beamforming methods. For STBC, define that the MMSE solutions for $x_{k,1,1}$ and $x_{k,2,1}$ are $\mathbf{w}_{k,1}$ and $\mathbf{w}_{k,2}$, respectively. Then we can have:

$$\mathbf{w}_{k,1} = \frac{\Phi_{(STBC,1)}^{-1}(k)\overline{\mathbf{H}}_{kk,1}}{\overline{\mathbf{H}}_{kk,1}^{H}\Phi_{(STBC,1)}^{-1}(k)\overline{\mathbf{H}}_{kk,1}} \tag{25}$$

$$\mathbf{w}_{k,2} = \frac{\Phi_{(STBC,2)}^{-1}(k)\overline{\mathbf{H}}_{kk,2}}{\overline{\mathbf{H}}_{kk,2}^{H}\Phi_{(STBC,2)}^{-1}(k)\overline{\mathbf{H}}_{kk,2}} \tag{26}$$

$$\Phi_{(STBC,1)}(k) = \sum_{i=1,i\neq k}^{K}\sum_{l=1}^{2}P_i(l)\rho_{ki}\overline{\mathbf{H}}_{ki,l}\overline{\mathbf{H}}_{ki,l}^{H} + P_k(2)\rho_{kk}\overline{\mathbf{H}}_{kk,2}\overline{\mathbf{H}}_{kk,2}^{H} + \sigma_N^2\mathbf{I}_{\mathrm{M}} \tag{27}$$

$$\Phi_{(STBC,2)}(k) = \sum_{i=1,i\neq k}^{K} \sum_{l=1}^{2} P_i(l)\rho_{ki}\overline{\mathbf{H}}_{ki,l}\overline{\mathbf{H}}_{ki,l}^{H} + P_k(1)\rho_{kk}\overline{\mathbf{H}}_{kk,1}\overline{\mathbf{H}}_{kk,1}^{H} + \sigma_N^2\mathbf{I}_{\mathrm{M}} \quad (28)$$

For Transmit Beamforming, the MMSE solution is:

$$\mathbf{w}_k = \frac{\Phi_{(BF)}^{-1}(k)\mathbf{H}_{kk}\mathbf{u}_k}{(\mathbf{H}_{kk}\mathbf{u}_k)^H\Phi_{(BF)}^{-1}(k)\mathbf{H}_{kk}\mathbf{u}_k} \quad (29)$$

$$\Phi_{(BF)}(k) = \sum_{i=1,i\neq k}^{K} P_i\rho_{ki}(\mathbf{H}_{ki}\mathbf{u}_k)(\mathbf{H}_{ki}\mathbf{u}_k)^H + \sigma_N^2\mathbf{I}_M \quad (30)$$

## APPENDIX B

### PROOF OF THEOREM 1

Note that we have assumed in (18) that $\{P_k = 0, k \notin U_P\}$. The iteration equation (23) can be proved to be a standard function [23] for $\{P_k, k \in U_P\}$ via similar manner in [21]. Now assume that with power vector $\mathbf{P}$, $U_P$ is feasible. Set the initial power vector $\mathbf{P}^0 = \mathbf{P}$, and run the iteration process by $\mathbf{P}^{n+1} = m(\mathbf{P}^n)$. According to Lemma 1 in [23], $\mathbf{P}^n$ will convergence to the fixed point of mapping, which is $\widehat{\mathbf{P}} = m(\widehat{\mathbf{P}})$. With Monotonicity property in standard function, for $1 \leq k \leq K$, we have $\widehat{P}_k \leq P_k^0 \leq P_T$. Finally, it has been proved in [21] that power and weight vectors corresponding to the fixed point of mapping are all unique. Thus the theorem follows.

## REFERENCES

[1] K. Sundaresan, R. Sivakumar, M. A. Ingram, and T.-Y. Chang, "A fair medium access control protocol for ad-hoc networks with MIMO links," *Proc. IEEE INFOCOM'04*, vol. 4, pp. 2559–2570, Mar. 7–11, 2004.

[2] B. Hamdaoui and P. Ramanathan, "A cross-layer admission control framework for wireless ad-hoc networks using multiple antennas," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 11, pp. 4014–4024, November 2007.

[3] J.-S. Park, A. Nandan, M. Gerla, and H. Lee, "SPACE-MAC: enabling spatial reuse using MIMO channel-aware MAC," in *Proc. IEEE ICC '04*, vol. 5, May 16–20, 2005, pp. 3642–3646.

[4] A. Singh, P. Ramanathan, and B. Van Veen, "Spatial reuse through adaptive interference cancellation in multi-antenna wireless networks," in *Proc. IEEE Globecom '05*, vol. 5, Nov. 28–Dec. 2, 2005.

[5] M. Park, S.-H. Choi, and S. Nettles, "Cross-layer mac design for wireless networks using mimo," *Global Telecommunications Conference, 2005. GLOBECOM '05. IEEE*, vol. 5, pp. 5 pp.–, Nov.-2 Dec. 2005.

[6] M. Zorzi, J. Zeidler, A. Anderson, B. Rao, J. Proakis, A. L. Swindlehurst, M. Jensen, and S. Krishnamurthy, "Cross-layer issues in MAC protocol design for MIMO ad hoc networks," *IEEE Wireless Commun. Mag.*, vol. 13, no. 4, pp. 62–76, Aug. 2006.

[7] R. Bhatia and L. Li, "Throughput optimization of wireless mesh networks with mimo links," *INFOCOM 2007. 26th IEEE International Conference on Computer Communications. IEEE*, pp. 2326–2330, May 2007.

[8] Y. Xing, C. N. Mathur, M. Haleem, R. Chandramouli, and K. Subbalakshmi, "Dynamic spectrum access with qos and interference temperature constraints," *Mobile Computing, IEEE Transactions on*, vol. 6, no. 4, pp. 423–433, April 2007.

[9] *Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification*, IEEE Std. 802.11, 1997.

[10] A. PAULRAJ, D. GORE, R. NABAR, and H. BOLCSKEI, "An overview of mimo communications - a key to gigabit wireless," *Proceedings of the IEEE*, vol. 92, no. 2, pp. 198–218, Feb 2004.

[11] S. Ye and R. Blum, "On the rate regions for wireless mimo ad hoc networks," *Vehicular Technology Conference, 2004. VTC2004-Fall. 2004 IEEE 60th*, vol. 3, pp. 1648–1652 Vol. 3, Sept. 2004.

[12] F. Ye, S. Yi, and B. Sikdar, "Improving spatial reuse of ieee 802.11 based ad hoc networks," *Global Telecommunications Conference, 2003. GLOBECOM '03. IEEE*, vol. 2, pp. 1013–1017 Vol.2, Dec. 2003.

[13] T. S. Rappaport, Ed., *Wireless Communications: Principles & Practice.* Englewood Cliffs, NJ: Prentice-Hall, 1996.

[14] A. Goldsmith, Ed., *Wireless communications*. Cambridge ; New York: Cambridge University Press, 2005.

[15] S. Alamouti, "A simple transmit diversity technique for wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.

[16] B. Chen and M. J. Gans, "MIMO communications in ad hoc networks," *IEEE Trans. Signal Processing*, vol. 54, no. 7, pp. 2773–2783, July 2006.

[17] M. McKay, I. Collings, A. Forenza, and R. Heath, "Multiplexing/beamforming switching for coded mimo in spatially correlated channels based on closed-form ber approximations," *Vehicular Technology, IEEE Transactions on*, vol. 56, no. 5, pp. 2555–2567, Sept. 2007.

[18] U. Madhow and M. Honig, "Mmse interference suppression for direct-sequence spread-spectrum cdma," *Communications, IEEE Transactions on*, vol. 42, no. 12, pp. 3178–3188, Dec 1994.

[19] R. Chen, J. Andrews, R. Heath, and A. Ghosh, "Uplink power control in multi-cell spatial multiplexing wireless systems," *Wireless Communications, IEEE Transactions on*, vol. 6, no. 7, pp. 2700–2711, July 2007.

[20] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[21] F. Rashid-Farrokhi, L. Tassiulas, and K. J. R. Liu, "Joint optimal power control and beamforming in wireless networks using antenna arrays," *IEEE Trans. Commun.*, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.

[22] R. D. Yates and C.-Y. Huang, "Integrated power control and base station assignment," *IEEE Trans. Veh. Technol.*, vol. 44, no. 3, pp. 638–644, Aug. 1995.

[23] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 1341–1347, Sept. 1995.

[24] S. S. Ellis Horowitz, Ed., *Fundamentals of Computer Algorithms*. Rockville, Md.: Computer Science Press, 1984.

TABLE I

PARAMETERS IN THE SIMULATIONS

| Parameter | Value |
|---|---|
| Path Loss | $d_0 = 1\text{m}$ |
| | $L_P(d_0) = 46\text{dB}$ |
| | Exponent factor $\alpha = 3$ |
| Noise Power Spectral Density | $\eta_n = -174\text{dBm/Hz}$ |
| Noise Figure | $F_n = 4\text{dB}$ |
| Bandwidth | $W = 1\text{MHz}$ |
| Maximum Transmit Power | $P_T = 20\text{dBm}$ |
| SINR threshold | $\gamma_T = 10\text{dB}$ |

TABLE II

PARAMETERS IN THE APPROXIMATION RESULTS OF AVERAGE MNCL WITH TRANSMIT BEAMFORMING

| $M$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $a_1$ | 0 | 0.0040 | $-0.0003$ | 0.0030 |
| $b_1$ | 0.9680 | 0.9950 | 0.9990 | 0.9956 |
| $a_2$ | 0.9961 | 1.9587 | 2.6045 | 3.5086 |
| $b_2$ | 0.1084 | 0.2121 | 0.3013 | 0.3171 |
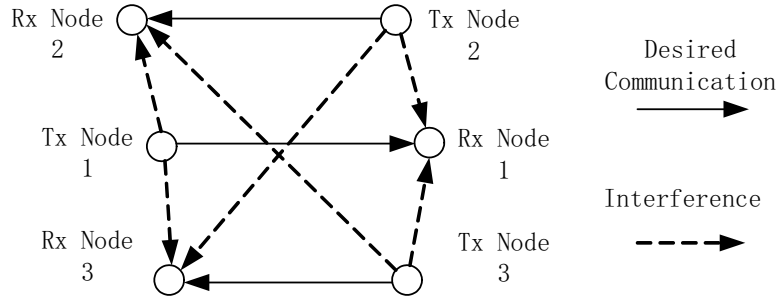


Fig. 1. Illustration of the considered network. Solid lines are the desired communication, while dashed lines are the interference.
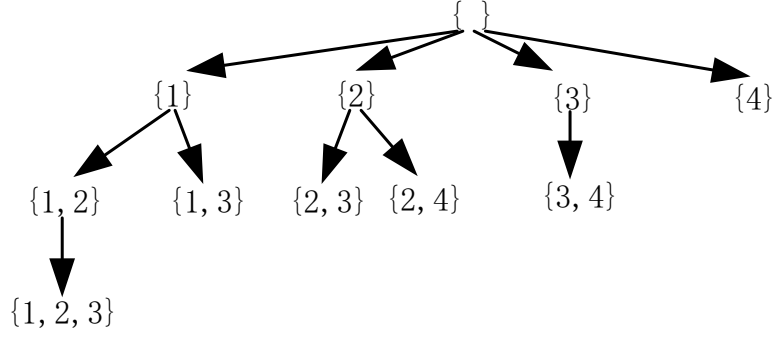
Fig. 2. Tree structure for backtracking search.
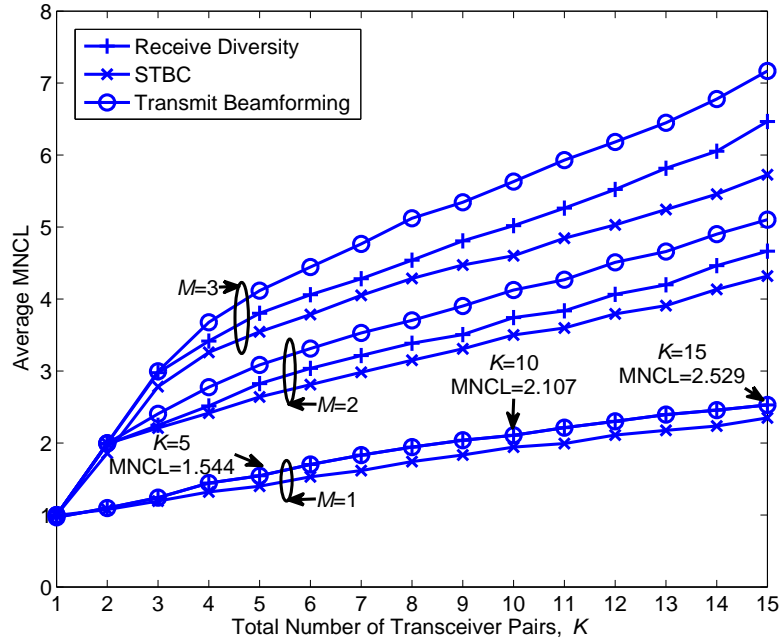


Fig. 3. Average MNCL under different MIMO techniques. $M$ is from 1 to 3.
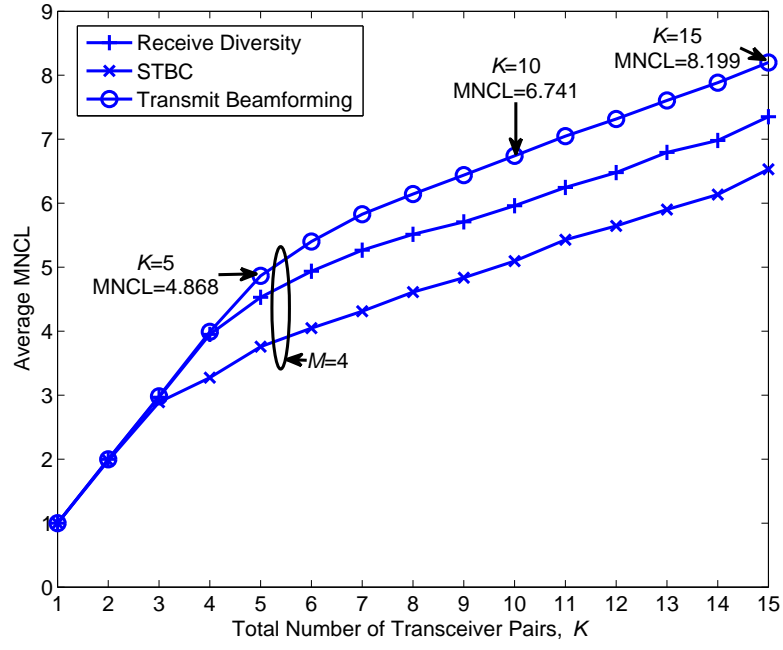
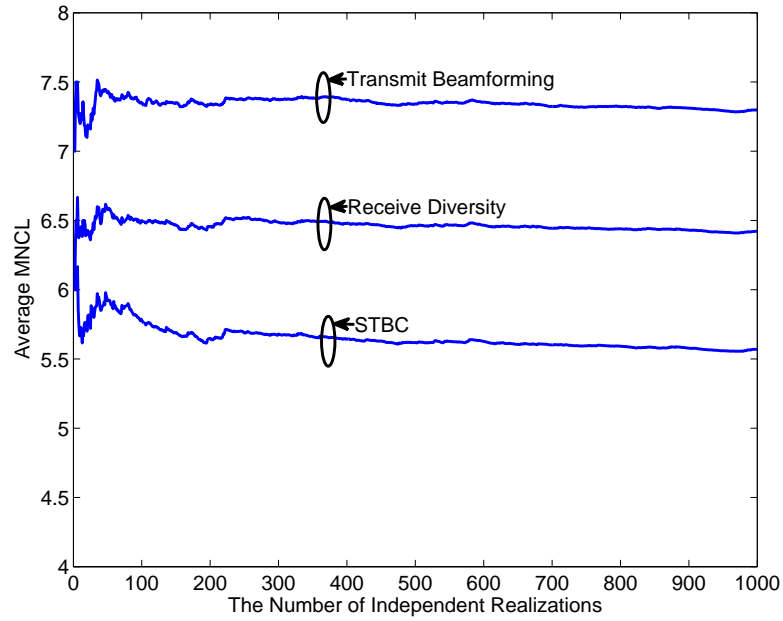Fig. 4. Average MNCL under different MIMO techniques. $M$ is set as 4.



Fig. 5. Estimate results for the MNCL versus the number of independent realizations. Assume total 12 transceiver pairs and $M = 4$.
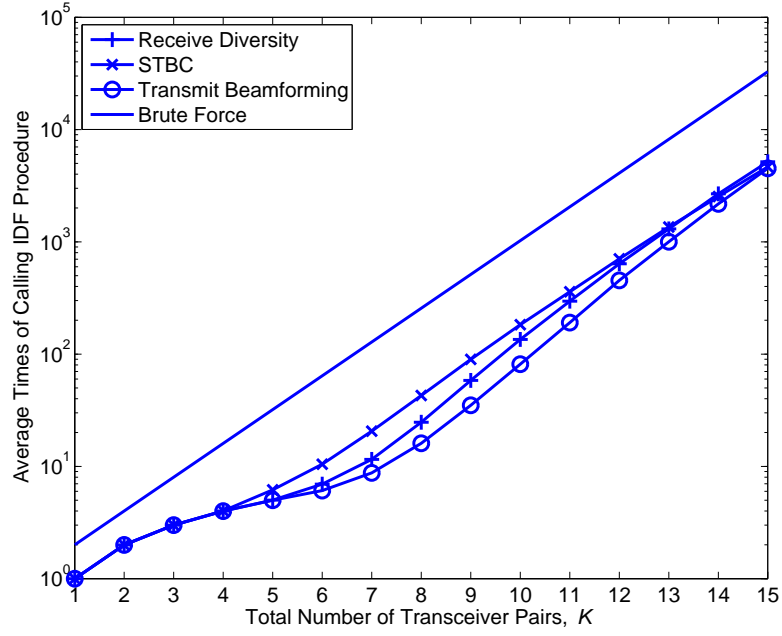
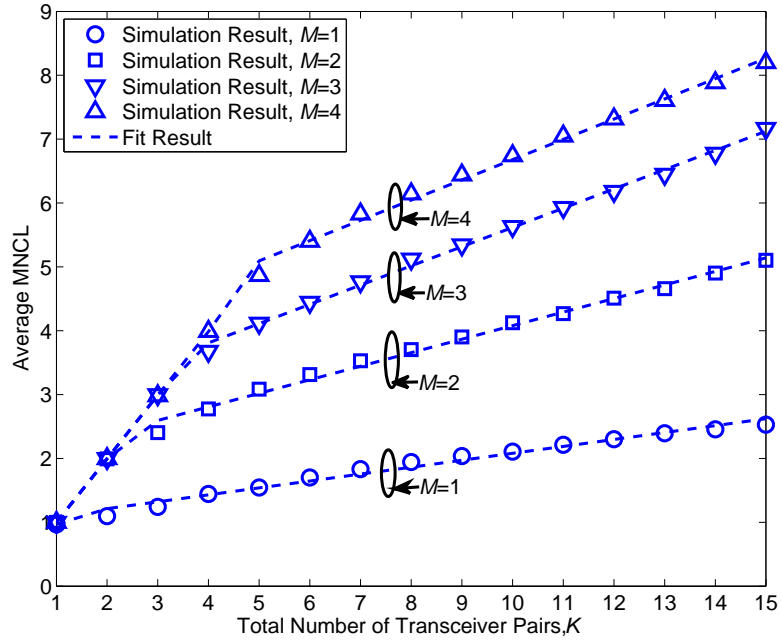Fig. 6.   Average number of times the IDF procedure was called versus the number of transceiver pairs for $M = 4$.



Fig. 7.   Simulation and approximation results for average MNCL with Transmit Beamforming. Assume 12 pairs equipped with 4 receive antennas.
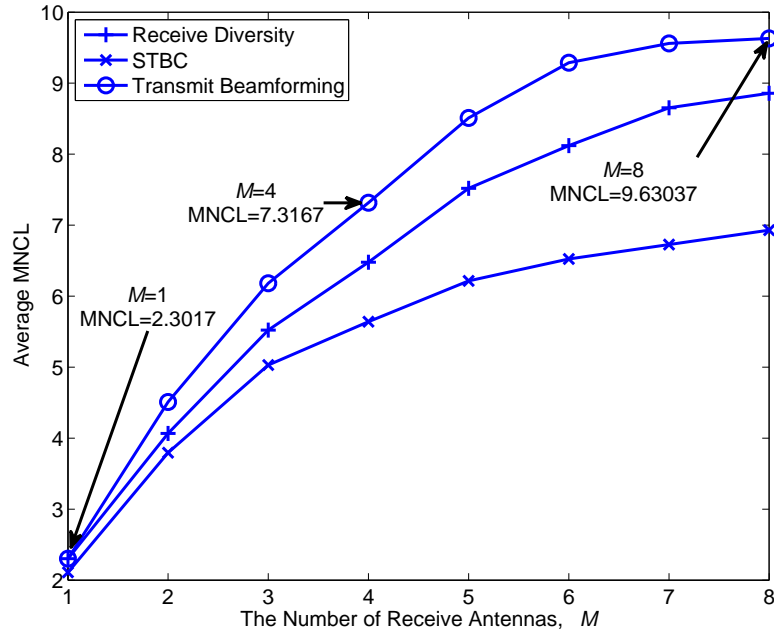
Fig. 8.   Results under different numbers of receive antennas. Assume total 12 pairs. Up to 8 receive antennas are simulated.
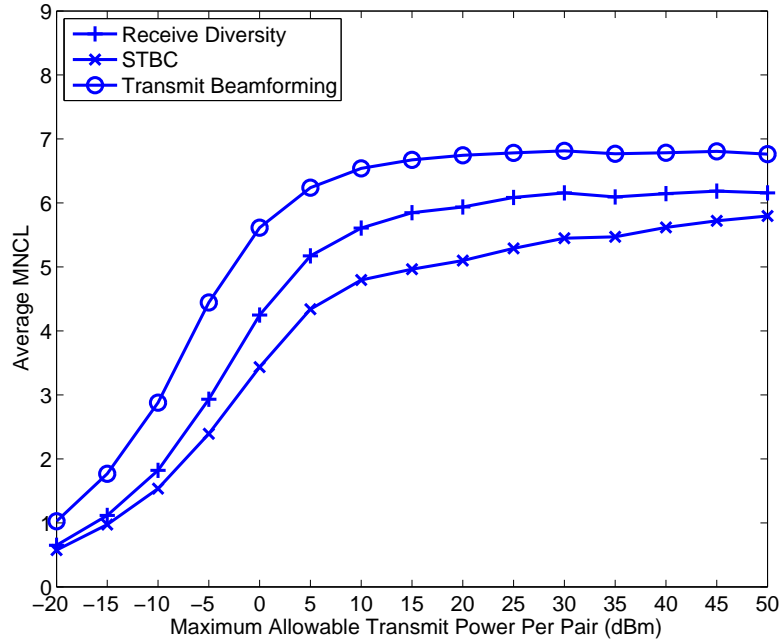


Fig. 9.   Average MNCL versus different maximum allowable transmit power per pair. Assume a total of 10 pairs, $M$ is fixed as 4.